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Interactions among Finite Rectangular Faults in a Viscoelastic Half-Space

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Abstract:*Stress accumulation near earthquake generating fault system during the aseismic period in a seismically active region becomes a subject of research during the last few decades. Mathematical models have been formulated to study the eﬀect on the nature of stress accumulation due to interactions of neighbouring faults. Two interacting, finite strike-slip faults, situated in a viscoelastic half-space representing the Lithosphere-Asthenosphere system, is considered here. The strikes of the faults are not parallel here. Stresses and strain accumulation in the region due to various tectonic processes, such as mantle convection and plate movements etc. ultimately tends to movements across the faults. In the present paper, analytical expressions for displacements, stresses and strain have been obtained using suitable mathematical techniques developed for this purpose. It is found that movement across one fault has considerable eﬀects on rate of stress accumulation near the other. A detailed study of these expressions may give some ideas about the nature of stress-strain accumulation in the system, which may be useful in formulating an effective earthquake prediction programme.*

Keywords:*Aseismic period, Lithosphere-Asthenosphere system, Finite fault, Stress accumulation, Viscoelastic half-space, Mantle convection, Earthquake prediction.*

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I. Introduction

Modeling of dynamical processes which leads to an earthquake is one of the main concerns in theoretical seismology at present. Two major seismic events are usually separated by a comparatively long aseismic periods of order of few decades or so. During the aseismic period slow and continuous surface movements are observed with the help of sophisticated measuring instruments. Such aseismic surface movements indicate that slow aseismic change of stress and strain are occurring in the region which may eventually lead to sudden or creeping movements across the seismic faults situated in the region. Modeling of aseismic ground deformation was carried out by a number of seismologists including Ghosh, et. al.[1], Chinnery[2,3], Karato[4], Cohen[5], Mukhopadhyay, et. al.[6,7], Piombo, et.al.[8], Mukhopadhyay, et. al.[9,10], Rosen and Singh[11], Sato[12], Segal[13], Sen, et. al.[14], Ghosh and Sen[15], Sen and Debnath[16,17], Debnath and Sen[18,19], Debnath[20], Debnath and Sen[21].They did a wonderful work in analyzing the displacement, stress and strain in the layered medium. In most of the earlier works elastic or viscoelastic layer or half-space medium were considered to represent the Lithosphere-Asthenosphere system. In most of the cases the faults were taken to be too long compared to its depth, so that the problem reduced to a 2D model. Noting that there are several faults which are not so long compared to their depth, a 3D model is more useful. In most of the theoretical models on finite faults developed so far, the strikes of the faults aretaken to be parallel. But fault system may often consist of faults of non-parallel strike. With these points in view, in the present paper we consider two non-parallel surface breaking strike-slip faults of finite length situated in a viscoelastic half-space. The medium is under the action of tectonic forces due to mantle convection or some related phenomena. It is assumed that the faults undergo a sudden movement when the stresses in the region near them exceed certain threshold values, which depend on the cohesive and frictional forces across the faults.

II. Formulation

We consider two rectangular strike-slip faults F_1 and F_2 of lengths $2L_1$ and $2L_2$ respectively in a viscoelastic half space of linear Maxwell type. The strike of the faults on the free surface are not parallel and making an angle θ. Let D_1 and D_2 be the width of the faults F_1 and F_2 respectively.

A rectangular Cartesian coordinate system is used for the fault F_1 with the mid-point O of the fault F_1 as the origin, the strike of the fault along the y_1 axis , y_2 axis perpendicular to the fault F_1 and y_3 axis pointing downwards so that the fault F_1 is given by $F_1{:}(-L_1 \le y_1 \le L_1,\ y_2 = 0,\ 0 \le y_3 \le D_1)$. Similarly for the fault F_2 we introduce a rectangular Cartesian coordinate system with the midpoint O′ of the fault F_2 as the origin. We take the coordinate of O′ as (d, D, 0) with respect to the coordinate system (y_1, y_2, y_3). We take the strike of the fault along the z_1 axis , z_2 axis perpendicular to the fault F_2 and z_3 axis pointing downwards so that the fault F_2 is given by F_2: ($-L_2 \le z_1 \le L_2$, $z_2 = 0$, $0 \le z_3 \le D_2$) as shown inFigure 1. y_3 and z_3 axis are parallel.

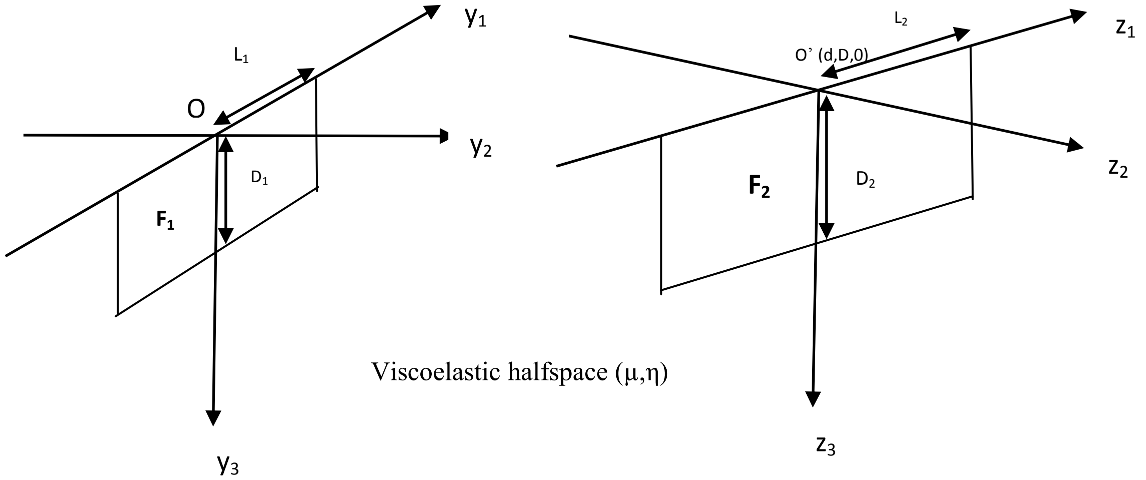

Figure 1 : Section of the model by the plane $y_1 = 0$

The relations between these two coordinate systems are given by:

$$z_1 = (y_1 - d)\cos\theta + (y_2 - D)\sin\theta,$$
$$z_2 = -(y_1 - d)\sin\theta + (y_2 - D)\cos\theta,$$
$$z_3 = y_3$$

For the fault F_1 let (u_i), (τ_{ij}) and (e_{ij}) be the components of displacements, stresses and strains [i, j = 1, 2, 3] .

The section of this model in the plane $y_1 = 0$ is shown in Figure1.

Stress−Strain relations (Constitutive equations): For the viscoelastic Maxwell type medium the constitutive equations are taken to be:

$$\left.\begin{aligned}
\left(\frac{1}{\eta}+\frac{1}{\mu}\frac{\partial}{\partial t}\right)\tau_{11} &= \frac{\partial^2 u_1}{\partial t\partial y_1}\\
\left(\frac{1}{\eta}+\frac{1}{\mu}\frac{\partial}{\partial t}\right)\tau_{12} &= \frac{1}{2}\frac{\partial}{\partial t}\left(\frac{\partial u_1}{\partial y_2}+\frac{\partial u_2}{\partial y_1}\right)\\
\left(\frac{1}{\eta}+\frac{1}{\mu}\frac{\partial}{\partial t}\right)\tau_{13} &= \frac{1}{2}\frac{\partial}{\partial t}\left(\frac{\partial u_1}{\partial y_3}+\frac{\partial u_3}{\partial y_1}\right)\\
\left(\frac{1}{\eta}+\frac{1}{\mu}\frac{\partial}{\partial t}\right)\tau_{22} &= \frac{\partial^2 u_2}{\partial t\partial y_2}\\
\left(\frac{1}{\eta}+\frac{1}{\mu}\frac{\partial}{\partial t}\right)\tau_{23} &= \frac{1}{2}\frac{\partial}{\partial t}\left(\frac{\partial u_2}{\partial y_3}+\frac{\partial u_3}{\partial y_2}\right)\\
\left(\frac{1}{\eta}+\frac{1}{\mu}\frac{\partial}{\partial t}\right)\tau_{33} &= \frac{\partial^2 u_3}{\partial t\partial y_3}
\end{aligned}\right\} \quad (1)$$

where η is the eff ective viscosity and μ is the eff ective rigidity of the material.

Stress equation of motion: The stresses satisfy the following equations (assuming quasi-static deformation for which the inertia terms are neglected) and body forces do not change during our investigation:

$$\left.\begin{aligned}
\frac{\partial}{\partial y_1}(\tau_{11})+\frac{\partial}{\partial y_2}(\tau_{12})+\frac{\partial}{\partial y_3}(\tau_{13}) &= 0\\
\frac{\partial}{\partial y_1}(\tau_{21})+\frac{\partial}{\partial y_2}(\tau_{22})+\frac{\partial}{\partial y_3}(\tau_{23}) &= 0\\
\frac{\partial}{\partial y_1}(\tau_{31})+\frac{\partial}{\partial y_2}(\tau_{32})+\frac{\partial}{\partial y_3}(\tau_{33}) &= 0
\end{aligned}\right\} \quad (2)$$

where $-\infty \le y_1 \le \infty,\ -\infty \le y_2 \le \infty,\ y_3 \ge 0,\ t \ge 0$

Boundary conditions: The boundary conditions are taken as, with t = 0 representing an instant when the medium is in aseismic state:

$$\left.\begin{aligned}&\lim_{y_1\to L_1-}\tau_{11}(y_1,\ y_2,\ y_3,\ t)=\lim_{y_1\to L_1+}\tau_{11}(y_1,\ y_2,\ y_3,\ t)=\tau_{L_1}(say)\\&\lim_{y_1\to -L_1-}\tau_{11}(y_1,\ y_2,\ y_3,\ t)=\lim_{y_1\to -L_1+}\tau_{11}(y_1,\ y_2,\ y_3,\ t)=\tau_{L_1}(say)\end{aligned}\right\}\quad(3)$$

for $y_2 = 0,\ 0 \le y_3 \le D_1,\ t \ge 0$

assuming that the stresses maintaining a constant value τ_{L_1} at the tip of the fault F_1 along y_1 axis the value of this constant stress is likely to be small enough so that no further extension is possible along the y_1 axis.

$\tau_{12}(y_1,\ y_2,\ y_3,\ t) \to \tau_\infty(t)$ (4)

as $|y_2| \to \infty,\ -\infty < y_1 < \infty,\ y_3 \ge 0,\ t \ge 0$

$\tau_\infty(t)$ is the stress in the strike-slip direction for F_1 maintained by the tectonic forces due to mantle convection and other related phenomena. It is assumed to be slowly increasing with time and is the main driving force for a movement across F_1 in the strike-slip direction.

On the free surface $y_3 = 0\ (-\infty < y_1 < \infty,\ -\infty < y_2 < \infty\ ,\ t \ge 0)$

$$\left.\begin{aligned}&\tau_{13}(y_1,\ y_2,\ y_3,\ t)=0\\&\tau_{23}(y_1,\ y_2,\ y_3,\ t)=0\\&\tau_{33}(y_1,\ y_2,\ y_3,\ t)=0\end{aligned}\right\}\quad(5)$$

Also as $y_3 \to \infty,\ -\infty < y_1, y_2 < \infty,\ t \ge 0$

$$\left.\begin{aligned}&\tau_{13}(y_1,\ y_2,\ y_3,\ t)=0\\&\tau_{23}(y_1,\ y_2,\ y_3,\ t)=0\\&\tau_{33}(y_1,\ y_2,\ y_3,\ t)=0\end{aligned}\right\}\quad(6)$$

$$\tau_{22}(y_1,\ y_2,\ y_3,\ t)=0 \quad (7)$$

as $|y_2| \to \infty,\ -\infty < y_1 < \infty,\ y_3 \ge 0,\ t \ge 0$

Initial conditions: Let $(u_i)_0, (\tau_{ij})_0$ and $(e_{ij})_0$, [i, j = 1, 2, 3] be the values of (u_i), (τ_{ij}) and (e_{ij}) at time t=0.

III. Displacements, stresses and strains in the absence of any fault movement

In the absence of any fault movement the displacements and stresses are continuous throughout the model. In order to obtain the expressions for displacement, strain and stresses we take Laplace transform of (1) to (7) with respect to t. The resulting boundary value problem can be solved easily. The solution obtained by taking inverse Laplace transforms, are given below:

$$\left.\begin{aligned}
u_1(y_1,\ y_2,\ y_3,\ t) &= (u_1)_0 + \frac{\tau_{L_1}}{\mu}y_1 t + \frac{y_2}{\mu}\left[\tau_\infty(t) - \tau_\infty(0) + \frac{\mu}{\eta}\int_0^t \tau_\infty(\tau)\,d\tau\right]\\
u_2(y_1,\ y_2,\ y_3,\ t) &= (u_2)_0 + \frac{y_1 + y_2}{\mu}\left[\tau_\infty(t) - \tau_\infty(0) + \frac{\mu}{\eta}\int_0^t \tau_\infty(\tau)\,d\tau\right]\\
u_3(y_1,\ y_2,\ y_3,\ t) &= (u_3)_0\\
\tau_{11}(y_1,\ y_2,\ y_3,\ t) &= (\tau_{11})_0 e^{-\frac{\mu t}{\eta}} + \frac{\mu}{\eta}\tau_{L_1}\left(1 - e^{-\frac{\mu t}{\eta}}\right)\\
\tau_{12}(y_1,\ y_2,\ y_3,\ t) &= \tau_\infty(t) - [\tau_\infty(0) - (\tau_{12})_0]e^{-\frac{\mu t}{\eta}}\\
\tau_{13}(y_1,\ y_2,\ y_3,\ t) &= (\tau_{13})_0 e^{-\frac{\mu t}{\eta}}\\
\tau_{22}(y_1,\ y_2,\ y_3,\ t) &= (\tau_{22})_0 e^{-\frac{\mu t}{\eta}}\\
\tau_{23}(y_1,\ y_2,\ y_3,\ t) &= (\tau_{23})_0 e^{-\frac{\mu t}{\eta}}\\
\tau_{33}(y_1,\ y_2,\ y_3,\ t) &= (\tau_{33})_0\\
e_{11}(y_1,\ y_2,\ y_3,\ t) &= (e_{11})_0 + \frac{\tau_{L_1} t}{\mu}\\
e_{12}(y_1,\ y_2,\ y_3,\ t) &= \frac{1}{2}(e_{12})_0 + \frac{1}{\mu}\left[\tau_\infty(t) - \tau_\infty(0) + \frac{\mu}{\eta}\int_0^t \tau_\infty(\tau)\,d\tau\right]
\end{aligned}\right\}\quad(8)$$

From the above solution we find that the stress component τ_{12} increases with time and tends to $\tau_\infty(t)$ as t tends to ∞, while τ_{22}, τ_{23} tends to zero, but τ_{33} remains constant value $(\tau_{33})_0$. We assume that the rheological properties of the half space near the faults are such that when the relevant stress component τ_{12} reaches a certain threshold value τ_{c_1} (say) after a time T_1 (say) the fault F_1 slips. The magnitude of slip is expected to satisfy the following conditions:

(C1) Its value will be maximum near the middle of the fault on the free surface.
(C2) Its will gradually decrease to zero at the tip of the fault F_1 along its length.
(C3) The magnitude of the slip will decrease with y_3 as we move downwards and ultimately tends to zero near the lower edge of the fault F_1.

If $f_1(y_1,\ y_3)$be the slip function, it should satisfy the above conditions.

IV. Displacements, stresses and strains after the commencement of the fault movement

We assume that after a time T_1, the stress component τ_{12} (which is the main driving force for the strike-slip motion of the fault) exceeds the critical value τ_{c_1}, the fault F_1 slips and the other fault F_2 remains locked. Then (1) – (7) are satisfied with the following condition of slip across F_1 :

$$[(u_1)]_{F_1} = U_1 . f_1(y_1,\ y_3).\ H(t_1) \qquad (9)$$

where$[(u_1)]_{F_1}$ = the discontinuity of u_1 across F_1 = $\lim_{y_2\to 0+}(u_1) - \lim_{y_2\to 0-}(u_1)$

$$(-L_1 \le y_1 \le L_1,\ y_2 = 0,\ 0 \le y_3 \le D_1),\ t_1 = t - T_1,\ t_1 \ge 0$$

and$H(t_1)$ is the Heaviside step function.

Taking Laplace transform of (9) we get,

$$[(\bar{u}_1)]_{F_1} = \frac{U_1}{p} . f_1(y_1,\ y_3), \qquad (10)$$

p being the Laplace transform variable.

We try to find the solution as:

$$\left.\begin{aligned} u_i &= (u_i)_1 + (u_i)_2 \\ \tau_{ij} &= (\tau_{ij})_1 + (\tau_{ij})_2 \\ e_{ij} &= (e_{ij})_1 + (e_{ij})_2 \end{aligned}\right\} \qquad (11)$$

where$(u_i)_1, (\tau_{ij})_1, (e_{ij})_1$ $[i,\ j = 1,\ 2,\ 3]$ are continuous everywhere in the model and given by (8).

For the second part we note that $(u_2)_2, (u_3)_2$ are both continuous even after the fault slip, so that $(u_2)_2 = (u_3)_2 = 0$ while $(u_1)_2$ satisfies the dislocation condition given by (9).

$(u_1)_2$satisfies 3D Laplace equation as :

$$\nabla^2(\bar{u}_1)_2 = 0 \qquad (12)$$

with the modified boundary condition

$$\bar{\tau}_{12}(y_1,\ y_2,\ y_3,\ t) \to 0 \qquad (13)$$

as$|y_2| \to \infty,\ -\infty < y_1 < \infty,\ y_3 \ge 0$

And the other boundary conditions are same as (3) – (7).

We solve the above boundary value problem by modified Green's function method following Maruyama[22, 23], Rybicki[24] and the correspondence principle.

Let $Q(y_1,\ y_2,\ y_3)$ be any point in the field and $P(x_1,\ x_2,\ x_3)$ be any point on the fault, then we have

$$(\bar{u}_1)_2(Q) = \iint_{F_1} [(\bar{u}_1)_2(P)].\ G(P,\ Q)\ dx_3 dx_1$$

$$= \iint_{F_1} \frac{U_1}{p} . f_1(x_1,\ x_3). G(P,\ Q)\ dx_3 dx_1$$

where G is the Green's function satisfying the above boundary value problem and

$$G(P,\ Q) = \frac{\partial}{\partial x_2} G_1(P,\ Q)$$

where$G_1(P,\ Q) = \frac{1}{2\pi}\left[\frac{1}{\{(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\}^{\frac{1}{2}}} + \frac{1}{\{(y_1-x_1)^2+(y_2-x_2)^2+(y_3+x_3)^2\}^{\frac{1}{2}}}\right]$

Therefore, $G(P,\ Q)=\frac{1}{2\pi}\left[\frac{(y_2-x_2)}{\{(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\}^{\frac{3}{2}}}+\frac{(y_2-x_2)}{\{(y_1-x_1)^2+(y_2-x_2)^2+(y_3+x_3)^2\}^{\frac{3}{2}}}\right]$

$$(\bar{u}_1)_2(Q)=\iint_{F_1}\frac{U_1}{2\pi p}.f_1(x_1,\ x_3).\left[\frac{(y_2-x_2)}{\{(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\}^{\frac{3}{2}}}+\frac{(y_2-x_2)}{\{(y_1-x_1)^2+(y_2-x_2)^2+(y_3+x_3)^2\}^{\frac{3}{2}}}\right]dx_3dx_1$$

Taking inverse Laplace transformation, we get

$$(u_1)_2=\frac{U_1}{2\pi}H(t-T_1)\ \phi_1(y_1,\ y_2,\ y_3)$$

We also have, $(\bar{\tau}_{11})_2=\frac{p}{\frac{1}{\eta}+\frac{p}{\mu}}\frac{\partial(\bar{u}_1)_2}{\partial y_1}$

After taking inverse Laplace transformation, we get

$$(\tau_{11})_2=\frac{\mu U_1}{2\pi}H(t-T_1)e^{-\frac{\mu(t-T_1)}{\eta}}\psi_1(y_1,\ y_2,\ y_3)$$

Similarly, $(\tau_{12})_2=\frac{\mu U_1}{4\pi}H(t-T_1)e^{-\frac{\mu(t-T_1)}{\eta}}\psi_2(y_1,\ y_2,\ y_3)$

$$(\tau_{13})_2=\frac{\mu U_1}{4\pi}H(t-T_1)e^{-\frac{\mu(t-T_1)}{\eta}}\psi_3(y_1,\ y_2,\ y_3)$$

$$(\tau_{22})_2=0$$

$$(\tau_{23})_2=0$$

$$(\tau_{33})_2=0$$

$$(e_{11})_2=\frac{U_1}{2\pi}H(t-T_1)\ \psi_1(y_1,\ y_2,\ y_3)$$

$(e_{12})_2=\frac{U_1}{4\pi}H(t-T_1)\ \psi_2(y_1,\ y_2,\ y_3)$ (14)

where $\phi_1(y_1,\ y_2,\ y_3)=$
$\int_{-L_1}^{L_1}\int_0^{D_1}f_1(x_1,\ x_3).\left[\frac{(y_2-x_2)}{\{(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\}^{\frac{3}{2}}}+\frac{(y_2-x_2)}{\{(y_1-x_1)^2+(y_2-x_2)^2+(y_3+x_3)^2\}^{\frac{3}{2}}}\right]dx_3dx_1$

$$\psi_1(y_1,\ y_2,\ y_3)=\int_{-L_1}^{L_1}\int_0^{D_1}3f_1(x_1,\ x_3).\left[-\frac{(y_2-x_2)(y_1-x_1)}{\{(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\}^{\frac{5}{2}}}-\frac{(y_2-x_2)(y_1-x_1)}{\{(y_1-x_1)^2+(y_2-x_2)^2+(y_3+x_3)^2\}^{\frac{5}{2}}}\right]dx_3dx_1$$

$\psi_2(y_1,\ y_2,\ y_3)=\int_{-L_1}^{L_1}\int_0^{D_1}f_1(x_1,\ x_3).\left[\frac{(y_1-x_1)^2-2(y_2-x_2)^2+(y_3-x_3)^2}{\{(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\}^{\frac{5}{2}}}-\frac{(y_1-x_1)^2-2(y_2-x_2)^2+(y_3+x_3)^2}{\{(y_1-x_1)^2+(y_2-x_2)^2+(y_3+x_3)^2\}^{\frac{5}{2}}}\right]dx_3dx_1$

$$\psi_3(y_1,\ y_2,\ y_3)=\int_{-L_1}^{L_1}\int_0^{D_1}3f_1(x_1,\ x_3).\left[-\frac{(y_2-x_2)(y_3-x_3)}{\{(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\}^{\frac{5}{2}}}-\frac{(y_2-x_2)(y_3+x_3)}{\{(y_1-x_1)^2+(y_2-x_2)^2+(y_3+x_3)^2\}^{\frac{5}{2}}}\right]dx_3dx_1$$

(15)

From the solution we find that the stress further accumulates due to the tectonic activities and stresses either accumulates or releases due to the movement across the fault F_1. We assume that the second fault F_2 slips after a time T_2 when the accumulated relevant stress near it exceeds the critical value τ_{c_2}(say).

The slip condition is characterize by:

$[(u_1)]_{F_2}=U_2.f_2(z_1,\ z_3).\ H(t_2)$ (16)

where $[(u_1)]_{F_2}$ = the discontinuity of u_1 across F_2 = $\lim_{z_2\to0+}(u_1)-\lim_{z_2\to0-}(u_1)$

$$(-L_2 \le z_1 \le L_2,\ z_2 = 0,\ 0 \le z_3 \le D_2),\ t_2 = t - T_2,\ t_2 \ge 0$$

and $H(t_2)$ is the Heaviside step function.

For a strike-slip movement across the fault F_2 the solutions for displacements, stresses and strains obtained in a similar way as:

$$u_1' = \frac{U_2}{2\pi} H(t - T_2)\phi_1'(z_1,\ z_2,\ z_3)$$
$$u_2' = 0$$
$$u_3' = 0$$
$$T_{11} = \frac{\mu U_2}{2\pi} H(t - T_2) e^{-\frac{\mu(t-T_2)}{\eta}} \psi_1'(z_1,\ z_2,\ z_3)$$
$$T_{12} = \frac{\mu U_2}{4\pi} H(t - T_2) e^{-\frac{\mu(t-T_2)}{\eta}} \psi_2'(z_1,\ z_2,\ z_3)$$
$$T_{13} = \frac{\mu U_2}{4\pi} H(t - T_2) e^{-\frac{\mu(t-T_2)}{\eta}} \psi_3'(z_1,\ z_2,\ z_3)$$
$$T_{22} = 0$$
$$T_{23} = 0$$
$$T_{33} = 0$$
$$E_{11} = \frac{U_2}{2\pi} H(t - T_2)\ \psi_1'(z_1,\ z_2,\ z_3)$$
$$E_{12} = \frac{U_2}{4\pi} H(t - T_2)\ \psi_2'(z_1,\ z_2,\ z_3) \quad (17)$$

where $\phi_1', \psi_1', \psi_2', \psi_3'$ have similar expressions as those of $\phi_1, \psi_1, \psi_2, \psi_3$ respectively as given in (15) and can be obtained from them on replacing $f_1(y_1,\ y_3), L_1, D_1, y_1, y_2, y_3$ by $f_2(z_1,\ z_3), L_2, D_2, z_1, z_2$ and z_3 respectively.

In terms of $(y_1,\ y_2,\ y_3)$ the final solution after the movement across the fault F_2 can be obtained as follows:

$$u_1 = (u_1)_1 + (u_1)_2 + (u_1)_3$$
$$u_2 = (u_2)_1 + (u_2)_2 + (u_2)_3$$
$$u_3 = (u_3)_1 + (u_3)_2 + (u_3)_3$$
$$\tau_{11} = (\tau_{11})_1 + (\tau_{11})_2 + (\tau_{11})_3$$
$$\tau_{12} = (\tau_{12})_1 + (\tau_{12})_2 + (\tau_{12})_3$$
$$\tau_{13} = (\tau_{13})_1 + (\tau_{13})_2 + (\tau_{13})_3$$
$$\tau_{22} = (\tau_{22})_1 + (\tau_{22})_2 + (\tau_{22})_3$$
$$\tau_{23} = (\tau_{23})_1 + (\tau_{23})_2 + (\tau_{23})_3$$
$$\tau_{33} = (\tau_{33})_1 + (\tau_{33})_2 + (\tau_{33})_3$$
$$e_{11} = (e_{11})_1 + (e_{11})_2 + (e_{11})_3$$
$$e_{12} = (e_{12})_1 + (e_{12})_2 + (e_{12})_3 \quad (18)$$

where $(u_i)_1, (\tau_{ij})_1, (e_{ij})_1$ are given by (8), $(u_i)_2, (\tau_{ij})_2, (e_{ij})_2$ are given by (14), $(u_i)_3, (\tau_{ij})_3, (e_{ij})_3$ and u_i', T_{ij}, E_{ij} given in (17) are connected by the relations : $[\mathrm{i},\ \mathrm{j} = 1,\ 2,\ 3]$

$$(u_1)_3 = u_1' \cos\theta - u_2' \sin\theta$$
$$(u_2)_3 = u_1' \sin\theta + u_2' \cos\theta$$
$$(u_3)_3 = 0$$
$$(\tau_{11})_3 = T_{11}\cos^2\theta - T_{12} \sin 2\theta + T_{22}\sin^2\theta$$
$$(\tau_{12})_3 = \frac{T_{11}}{2} \sin 2\theta + T_{12} \cos 2\theta - \frac{T_{22}}{2} \sin 2\theta$$
$$(\tau_{13})_3 = T_{13} \cos\theta - T_{23} \sin\theta$$
$$(\tau_{22})_3 = T_{11}\sin^2\theta + T_{12} \sin 2\theta + T_{22}\cos^2\theta$$
$$(\tau_{23})_3 = T_{13} \sin\theta + T_{23} \cos\theta$$
$$(\tau_{33})_3 = T_{33}$$
$$(e_{11})_3 = E_{11} \cos\theta - E_{12} \sin\theta$$
$$(e_{12})_3 = E_{11} \sin\theta + E_{12} \cos\theta$$

V. Numerical computations

Following Cathles[25], Aki and Rechards[26] and the recent studies on rheological behavior of crust and upper mantle by Clift.et. al.[27] the values to the model parameters are taken as:

$\mu = 3 \times 10^{11}\ dyn\ cm^{-2}$

$$\eta = 6.35 \times 10^{20}\ poise$$

$D_1 =$ Depth of the fault F_1 = 10 km.

$D_2 =$ Depth of the fault F_2 = 15 km. [Noting that the depth of all major earthquake faults are in between 10 – 15 km.]

$D = 15$ km.

$d = 15$ km.

$2L_1 =$ Length of the fault F_1 = 40 km.

$2L_2 =$ Length of the fault F_2 = 60 km.

We take $\tau_\infty(t) = \tau_\infty = 300$ bars [assumed to be constant in our numerical computations. Post seismic observations reveal that stresses released in major earthquakes are of the order of 200 bars, in extreme cases it may be 400 bars]

$(\tau_{12})_0 = 20$ bars

We take slip functions as :

$$f_1(y_1, y_3) = \left(1 - \frac{{y_1}^2}{{L_1}^2}\right)\left(1 - \frac{3{y_3}^2}{{D_1}^2} + \frac{3{y_3}^3}{{D_1}^3}\right)\left(\frac{D_1 - y_3}{D_1}\right)$$

$$f_2(z_1, z_3) = \left(1 - \frac{{z_1}^2}{{L_2}^2}\right)\left(1 - \frac{3{z_3}^2}{{D_2}^2} + \frac{3{z_3}^3}{{D_2}^3}\right)\left(\frac{D_2 - z_3}{D_2}\right)$$

$$U_1 = 100\ cm$$

$$U_2 = 50\ cm$$

θ is assumed to be $\frac{\pi}{6}$

We compute the following quantities:

$$u_1(y_1, y_2, y_3, t) - (u_1)_0 = \frac{\tau_{L_1}}{\mu} y_1 t + \frac{y_2}{\mu}\left[\tau_\infty(t) - \tau_\infty(0) + \frac{\mu}{\eta}\int_0^t \tau_\infty(\tau)\, d\tau\right] + \frac{U_1}{2\pi} H(t - T_1)\ \phi_1(y_1, y_2, y_3)$$

$$u_2(y_1, y_2, y_3, t) - (u_2)_0 = \frac{y_1 + y_2}{\mu}\left[\tau_\infty(t) - \tau_\infty(0) + \frac{\mu}{\eta}\int_0^t \tau_\infty(\tau)\, d\tau\right]$$

$$e_{12}(y_1, y_2, y_3, t) - \frac{1}{2}(e_{12})_0 = \frac{1}{\mu}\left[\tau_\infty(t) - \tau_\infty(0) + \frac{\mu}{\eta}\int_0^t \tau_\infty(\tau)\, d\tau\right] + \frac{U_1}{4\pi} H(t - T_1)\, \psi_2(y_1, y_2, y_3)$$

$$\tau_{11}(y_1, y_2, y_3, t) = (\tau_{11})_0 e^{-\frac{\mu t}{\eta}} + \frac{\mu}{\eta}\tau_{L_1}\left(1 - e^{-\frac{\mu t}{\eta}}\right) + \frac{\mu U_1}{2\pi} H(t - T_1) e^{-\frac{\mu(t - T_1)}{\eta}} \psi_1(y_1, y_2, y_3)$$

$$\tau_{12}(y_1, y_2, y_3, t) = \tau_\infty(t) - [\tau_\infty(0) - (\tau_{12})_0] e^{-\frac{\mu t}{\eta}} + \frac{\mu U_1}{4\pi} H(t - T_1) e^{-\frac{\mu(t - T_1)}{\eta}} \psi_2(y_1, y_2, y_3)$$

$$\tau_{13}(y_1, y_2, y_3, t) = (\tau_{13})_0 e^{-\frac{\mu t}{\eta}} + \frac{\mu U_1}{4\pi} H(t - T_1) e^{-\frac{\mu(t - T_1)}{\eta}} \psi_3(y_1, y_2, y_3)$$

VI. Results and discussion

Surface shear strain due to fault movement on the free surface:

The shear strain e_{12} at distances from the strike of the fault $y_1 = 5$ km. on the free surface is compared in three phases

(i) before any fault movement
(ii) after the movement across the fault F_1
(iii) after the movement across the fault F_2 which are shown in Figure 2.

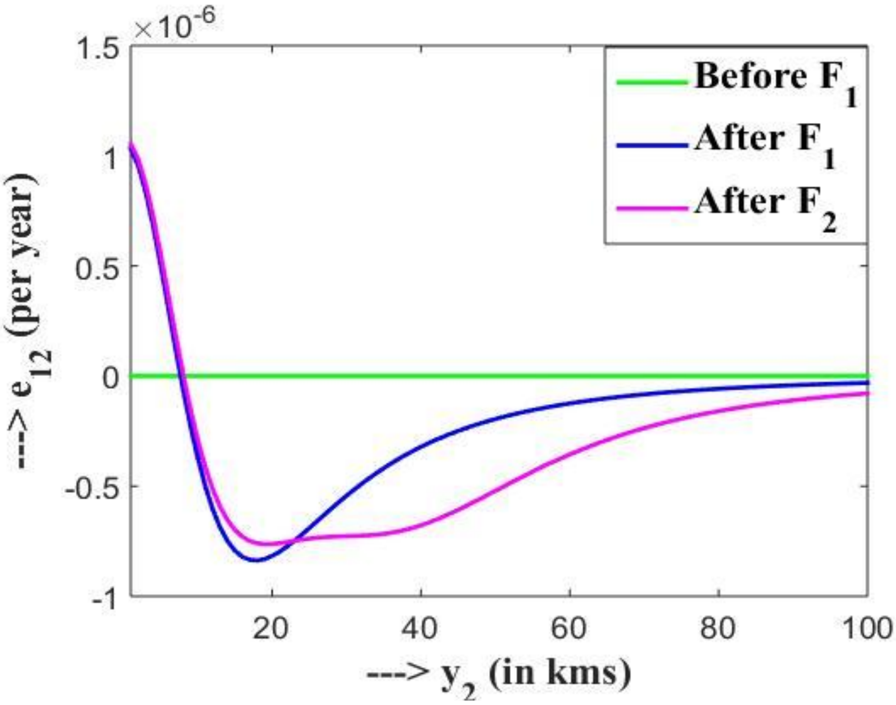

Figure 2: Variation of shear strain e_{12} on the free surface at$y_1 = 5$ km. with y_2 due to fault movements.

The magnitude of the surface shear strain due to the fault movement is found to be of order of $(1 - 2) \times 10^{-6}$ per year, which is conformity with the observed rate of shear strain accumulation during the aseismic period in seismically active regions. This established the validity of our model.

Displacement vectors on the free surface $y_3 = 0$ due to fault movement across F_1 and F_2 :

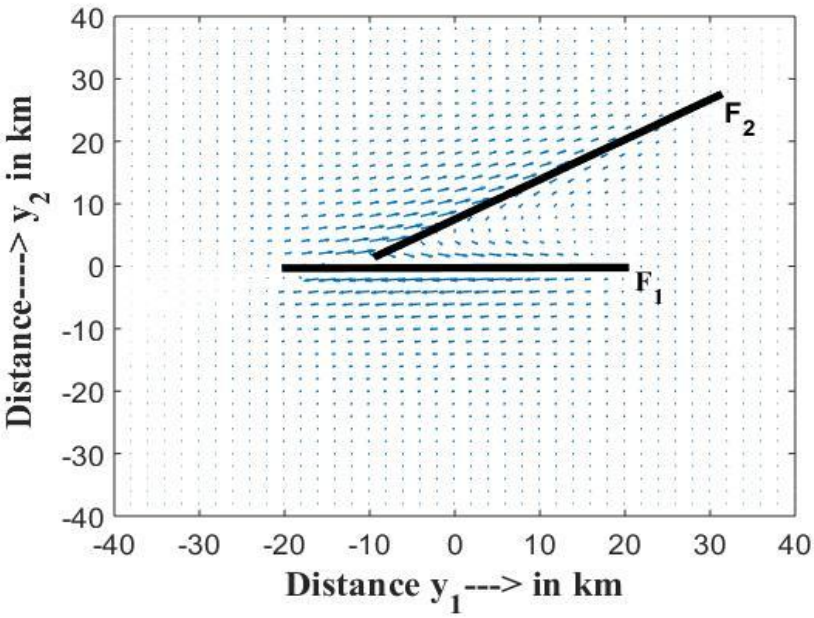

Figure 3: Displacement vector on the free surface after both the faults $F_1$and F_2 slips.

Figure 3 shows the displacement vectors on the free surface $y_3 = 0$ for $y_1 =$ - 40 km. to 40 km. and $y_2 =$ - 40 km. to 40 km. due to the fault movement across F_1 and F_2.

Accumulation of stress τ_{12} and T_{12} against time:

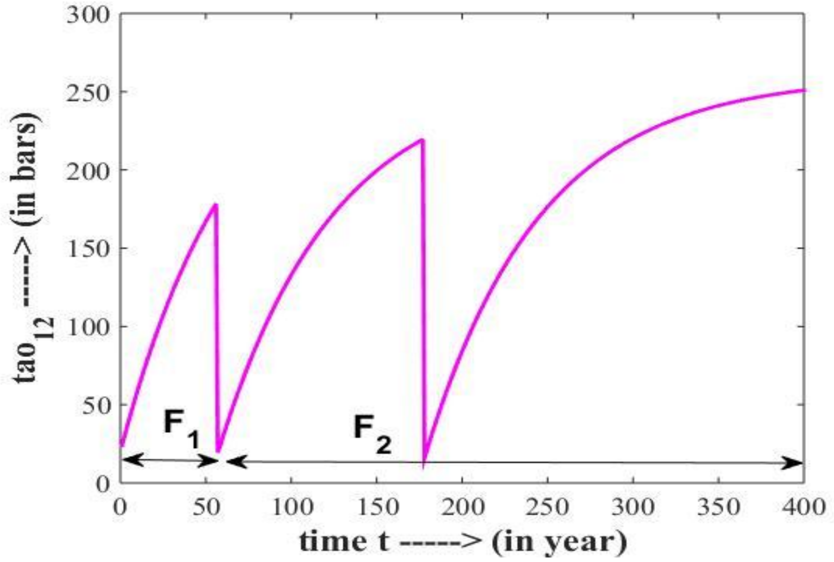

Figure 4: The accumulation of stress τ_{12} and T_{12} against time.

Figure 4 shows the accumulation of stress in three phases :
Phase I : $|\tau_{12}|$ near F_1 for $0 < t \leq T_1$,
Phase II : $|T_{12}|$ near F_2 for $T_1 < t \leq T_2$,
Phase III : $|T_{12}|$ near F_2 for $t \geq T_2$.

In each cases the stresses are found to be increasing but at a decreasing rate. Further the rate of increase of T_{12} is found to be less than that of τ_{12} on the avarage. During Phase III the rate of increase of accumulated stress is less than that of T_{12} during the second phase. In all the cases it has been assumed that the accumulated stresses have been released completely during the movement across F_1 and F_2 with the initial stress $(\tau_{12})_0 = 20$ bars and $(T_{12})_0 = 20$ bars at F_1 and F_2 respectively. It has been found that due to the movement across F_1 the magnitude of the stress changes slightly at $t = T_1$ (56 years). It starts accumulating and reaches a value τ_{c_2} (220 bars) at time $t = T_2$ (177 years). The movement across F_2 leads to a release in the accumulated stress and its magnitude undergo some changes from the initial stress $(T_{12})_0$ at $t = T_2$. It again starts accumulating and get passed τ_{c_2} at time $t = 309$ years, i.e. after a gap of about 132 years. The movement across F_2 has been enhanced by a considerable amount of time due to the joint effect of movements across F_1 and F_2.

Stress accumulation and release region due to fault movement across F_1 and F_2:

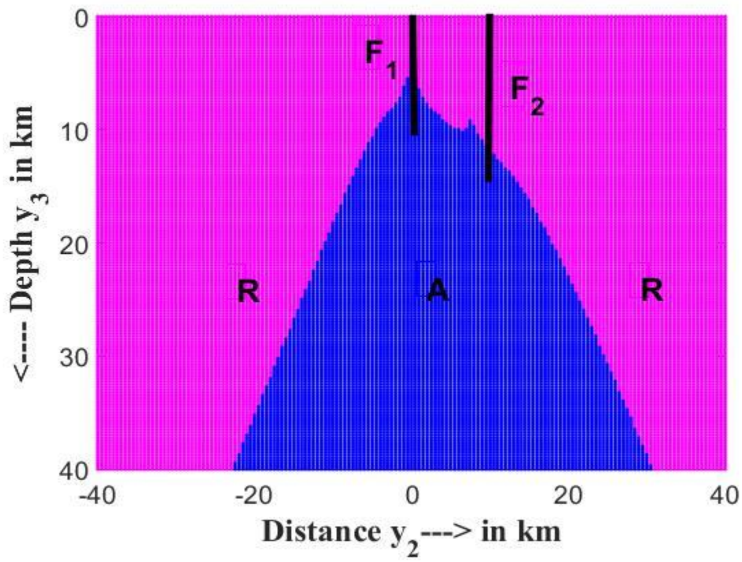

Figure 5: Region indication for stress accumulation and release due to the movement across both the faults F_1 and F_2.

Figure 5 shows the stress accumulation and release region due to the slipping movements across both the faults F_1 and F_2 for $t_2 = 1$ year over the region $y_1 = 0.5$ km., $y_2 =$ - 40 km. to 40 km. and $y_3 = 0$ km. to 40 km.

VII. Conclusion

i) In the above results we find that the strain on the free surface due to the movements of the faults is of the order of 10^{-6} per year.
ii) Displacement vectors on the two sides of the fault plane (on the free surface) are in opposite directions.
iii) We also found that the movements across one fault causes stress accumulation / release near the other fault which essentially depends on the dimensions of the faults as well as the distance between the faults.
iv) The shear and normal stress due to the fault movement sometimes get accumulated in certain region while there are some regions where the stress is found to get released due to the fault movement.
v) This approach may help us to understand the earthquake generating process to identify possible earthquake prediction.

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